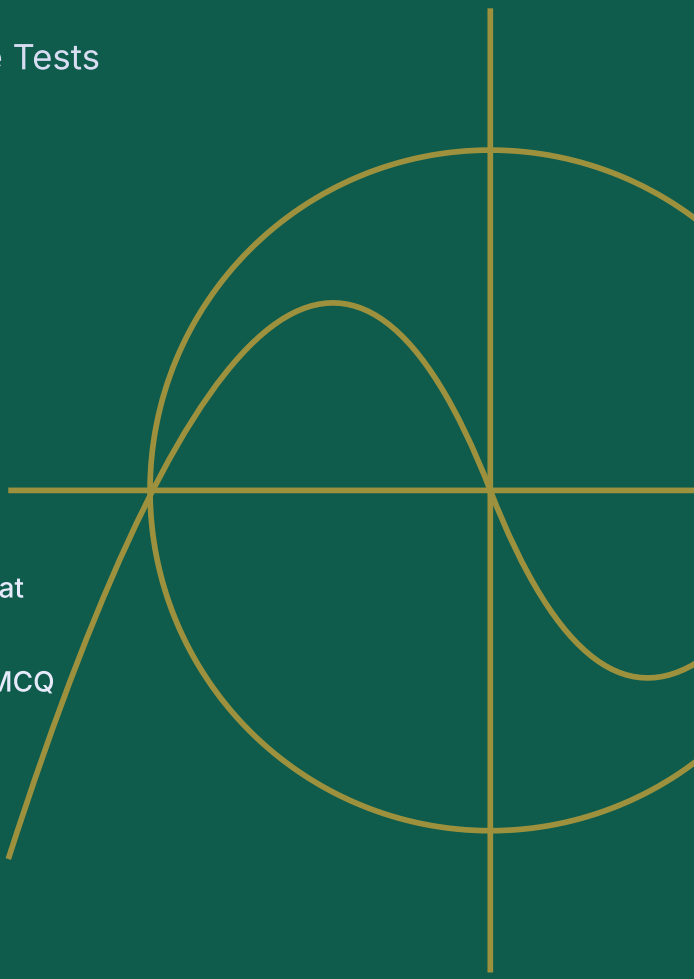
EXAM READY MATHEMATICS

Exam Ready Mathematics

GRADE 11

Study Guide, Worked Examples and Practice Tests for the Grade 11 Examination



- ✓ 8 units with clear key ideas, worked examples and “Watch out!” boxes
- ✓ 245 worked examples and 387+ practice questions at three levels
- ✓ 160 unit-quiz questions and 3 full mock exams (60 MCQ each)
- ✓ Diagnostic test · Formula Sheet · Top 20 Mistakes · teacher notes

Follows the units and order of the Ethiopian Grade 11 Mathematics textbook

ERMIAS WEDAJE TEGBARU

Exam Ready Mathematics

Grade 11 · Study Guide, Worked Examples and Practice Tests

By Ermias Wedaje Tegbaru
Civil engineer and author, Addis Ababa
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This is an independent study guide. It follows the units and order of the Ethiopian Grade 11 Mathematics student textbook, but it is not published or endorsed by the Ministry of Education. All explanations, examples and questions are original. Keep your textbook and your teacher's notes beside this book.

The answers were calculated and checked before printing, but no book is free of errors. If you find a mistake or disagree with an answer, please tell the author on Telegram (@Ermias_Wedaje) and it will be reviewed for the next edition.

How to Use This Book

Each unit follows the same pattern, so you always know where to look.

Key ideas
The rules and facts of the section, short and clear.

Worked examples
Step-by-step solutions. Cover the solution and try first.

⚠ Watch out!
The mistakes students make most often, and the right way.

Practice
Level 1 Basic · Level 2 Standard · Level 3 Challenge.

Unit quiz
20 multiple-choice questions in the style of the national examination.

For the Teacher
Timing, starters, misconceptions, activities and assessment.

For students

- Begin with the Diagnostic Test. It shows which units need the most work.
- Read the key ideas, then study each example with the solution covered.
- Do Level 1 and Level 2 practice for every section; try Level 3 when you are ready.
- Take each quiz in 30 minutes, mark it, and redo every question you got wrong.
- In the last weeks, learn the Key Facts Sheet and the Top 20 Mistakes page, and sit the three mock exams. Their answer keys show which unit each question comes from.

For teachers

- Each unit follows the textbook sections, so the book can sit beside your lessons.
- The “For the Teacher” page gives timing, starters, misconceptions and activities.
- The quizzes and mock exams can be used as class tests; every answer has a short reason.

For parents

- Ask your child to explain one worked example to you each day. Teaching is the best way to learn.
- Look at the quiz score after each unit, and celebrate progress, not only high marks.

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Diagnostic Test · Where Should I Start?

14 questions · 15 minutes. Take this test before you start the book. There are two questions on each unit. Your score only shows you where to spend your time.

1. If $f(x) = 4x + 3$, then $f^{-1}(11) =$

A. 2 B. 1/47 C. 47 D. 8

2. A business has cost $C(x) = 20x + 1000$ and revenue $R(x) = 60x$. The break-even quantity is $x =$

A. 25/2 B. 40000 C. 50/3 D. 25

3. Solve $18/x = 3$.

A. 9 B. 18 C. 6 D. 3

4. The solution of $(x - 3)/(x + 2) \geq 0$ is

A. $x < -2$ or $x \geq 3$ B. $x \leq -2$ or $x \geq 3$

C. $x \geq 3$ D. $-2 < x \leq 3$

5. The matrix $M = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$ has $\det M = 1$. The entry in row 1, column 1 of M^{-1} is

A. 1/3 B. −3 C. 1 D. 3

6. The rank of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is

A. 2 B. 1 C. 0 D. −1

7. For a 3×3 matrix A with $\det A = 3$, $\det(2A) =$

A. 6 B. 12 C. 24 D. 18

8. The determinant of $\begin{bmatrix} 5 & 6 \\ 5 & 1 \end{bmatrix}$ is

A. −25 B. 25 C. 35 D. 19

9. The midpoint of A(2, 3) and B(4, 5) is

A. (4, 5) B. (4, 4) C. (2, 2) D. (3, 4)

10. The magnitude of (3, 4) is

A. 1 B. 12 C. 7 D. 5

11. The matrix $\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ maps a figure of area 10 to a figure of area

A. 18 B. 20 C. 40 D. 80

12. Rotating (5, -1) through 90° anticlockwise about the origin gives

A. (−5, 1) B. (1, 5) C. (5, 1) D. (−1, −5)

13. The mean of 2, 2, 2, 9, 12, 2 is

A. 29/6 B. 35/6 C. 2 D. 12

14. The range of 6, 10, 15, 7 is

A. 9 B. 13 C. 15 D. 4

Example 1.2 — Relation from a rule

Let R = {(x, y) : y = x + 2, x in {1, 2, 3, 4}}. List R and give its domain and range.

Solution. R = {(1,3), (2,4), (3,5), (4,6)}. Domain = {1, 2, 3, 4}. Range = {3, 4, 5, 6}.

Example 1.3 — Count the relations

A = {1, 2} and B = {x, y}. How many relations are there from A to B?

Solution. $|A \times B| = 4$, so there are $2^4 = \mathbf{16}$ relations.

Example 1.4 — Exam Booster · Domain and range from a rule

Find the domain and range of the relation $y = \sqrt{(x - 1)}$.

Solution. We need $x - 1 \geq 0$, so the domain is $\mathbf{x \geq 1}$. A square root is never negative, so the range is $\mathbf{y \geq 0}$.

Example 1.5 — Exam Booster · Reading a graph

A relation is R = {(−1, 2), (0, 3), (2, 3), (4, −1)}. Give its domain and range.

Solution. Domain = {−1, 0, 2, 4}. Range = {−1, 2, 3} (3 is written once).

⚠ Watch out!

× Wrong	✓ Right
(1, 2) and (2, 1) are the same pair.	The order matters, so (1, 2) ≠ (2, 1).
The range lists every second element, even repeated ones, many times.	List each element of the range once.
A × B and B × A are equal.	In general A × B ≠ B × A (but they have the same number of elements).
The domain of $\sqrt{(x - 1)}$ is $x \leq 1$.	The expression under the root must be ≥ 0, so $x \geq 1$.

Practice 1.1

Level 1 — Basic

1. A = {1, 2}, B = {3, 4, 5}. How many elements has A × B?
2. Write the domain of {(1,4), (2,5), (3,6)}.
3. Write the range of {(1,4), (2,4), (3,6)}.
4. Is {(1,2), (3,4)} a relation from {1,3} to {2,4}?

Level 2 — Standard

5. List R = {(x, y) : y = 2x − 1, x in {0, 1, 2, 3}}.
6. A has 3 elements and B has 2. How many relations are there from A to B?
7. Find the domain of $y = \sqrt{(2x + 6)}$.

8. Find the range of $y = x^2$ for x in $\{-2, -1, 0, 1, 2\}$.

Level 3 — Challenge

9. Find the domain of $y = 1/(x - 3)$.
10. Let $A = \{1, 2, 3\}$. List the relation "$x < y$" on A.

1.2 Inverse of a Relation and Its Graph

Key ideas

- The **inverse** of a relation R is $R^{-1} = \{(b, a) : (a, b) \text{ in } R\}$: swap the elements of every pair.
- The domain of R^{-1} is the range of R, and the range of R^{-1} is the domain of R.
- To find the inverse of a relation given by an equation, **replace x by y and y by x**, then solve for y.
- The graph of R^{-1} is the **reflection of the graph of R in the line y = x**. If (a, b) lies on R, then (b, a) lies on R^{-1}.
- The inverse of a relation always exists (it may not be a function).

Example 1.6 — Inverse of a set of pairs

$R = \{(1,2), (3,4), (5,6)\}$. Find R^{-1}.

Solution. $R^{-1} = \{(2,1), (4,3), (6,5)\}$.

Example 1.7 — Inverse of an equation

Find the inverse of $y = 2x + 3$.

Solution. Swap: $x = 2y + 3$. Solve: $2y = x - 3$, so $\mathbf{y = (x - 3)/2}$.

Example 1.8 — Domain and range

$R = \{(x, y) : y = x^2, x \text{ in } \{-2, -1, 0, 1\}\}$. List R^{-1} and give its domain and range.

Solution. $R = \{(-2,4), (-1,1), (0,0), (1,1)\}$. So $R^{-1} = \{(4,-2), (1,-1), (0,0), (1,1)\}$. Domain = {0, 1, 4}, range = $\{-2, -1, 0, 1\}$.

Example 1.9 — Exam Booster · Reflection

The point (2, 5) is on the graph of R. Name a point on the graph of R^{-1}.

Solution. (5, 2).

Example 1.10 — Exam Booster · Inverse of a parabola

Find the inverse relation of $y = x^2$ and say whether it is a function.

Solution. Swap: $x = y^2$, so $y = \pm\sqrt{x}$. One x gives two values of y, so the inverse is **not a function**.

⚠ Watch out!

× Wrong	✓ Right
The inverse of $y = 2x + 3$ is $y = 1/(2x + 3)$.	$1/(2x + 3)$ is the reciprocal. The inverse swaps x and y: $y = (x - 3)/2$.
The graph of the inverse is the reflection in the x-axis.	It is the reflection in the line $y = x$.
The inverse of a relation is always a function.	Not always: the inverse of $y = x^2$ is $y = \pm\sqrt{x}$, not a function.

Practice 1.2

Level 1 — Basic

1. Write R^{-1} for $R = \{(1,5), (2,6)\}$.
2. Which line is the mirror between a graph and its inverse?
3. The domain of R is $\{1, 2\}$. What is the range of R^{-1}?

Level 2 — Standard

4. Find the inverse of $y = x + 5$.
5. Find the inverse of $y = 3x - 6$.
6. $(3, -2)$ is on R. Name a point on R^{-1}.

Level 3 — Challenge

7. Find the inverse of $y = 2x/(x + 1)$ as an expression.
8. Is the inverse of $\{(1,2), (3,2)\}$ a function?

1.3 Functions, Domain and Range

Key ideas

- A **function** from A to B is a relation in which **each element of A has exactly one image** in B. We write $y = f(x)$.
- **Vertical line test:** a graph is a function if no vertical line cuts it more than once.
- **One-to-one (injective):** different inputs give different outputs. **Horizontal line test:** no horizontal line cuts the graph more than once. **Onto (surjective):** every element of the codomain is an image.
- **Domain** = all allowed inputs. For polynomials: all real numbers. For fractions: exclude zeros of the denominator. For square roots: the expression under the root must be ≥ 0.
- **Range** = all outputs. For $y = a(x - h)^2 + k$: range $y \geq k$ if $a > 0$, and $y \leq k$ if $a < 0$.
- **Absolute value:** $|x| = x$ if $x \geq 0$ and $-x$ if $x < 0$. **Piecewise functions** use different rules on different intervals.

Example 1.11 — Evaluate

If $f(x) = x^2 - 4x + 1$, find $f(3)$ and $f(-1)$.

Solution. $f(3) = 9 - 12 + 1 = \mathbf{-2}$. $f(-1) = 1 + 4 + 1 = \mathbf{6}$.

Example 2.1 — Simplify

Simplify $(x^2 - 9)/(x^2 - x - 6)$ and state the restrictions.

Solution. $(x - 3)(x + 3)/((x - 3)(x + 2))$ = **$(x + 3)/(x + 2)$**, with $x \neq 3$ and $x \neq -2$.

Example 2.2 — Multiply

Simplify $(x^2 - 1)/(x + 2) \times (x^2 + 2x)/(x - 1)$.

Solution. $= (x - 1)(x + 1)/(x + 2) \times x(x + 2)/(x - 1)$ = **$x(x + 1)$**, with $x \neq -2, 1$.

Example 2.3 — Divide

Simplify $(x^2 - 4)/(x + 3) \div (x - 2)/(x^2 - 9)$.

Solution. $= (x - 2)(x + 2)/(x + 3) \times (x - 3)(x + 3)/(x - 2)$ = **$(x + 2)(x - 3)$**, with $x \neq -3, 2, 3$.

Example 2.4 — Add

Add $1/(x - 1) + 2/(x + 1)$.

Solution. LCD = $(x - 1)(x + 1)$. Numerator: $(x + 1) + 2(x - 1) = 3x - 1$. Result **$(3x - 1)/(x^2 - 1)$**.

Example 2.5 — Exam Booster · Subtract

Simplify $x/(x - 2) - 4/(x^2 - 4)$.

Solution. LCD = $(x - 2)(x + 2)$. Numerator: $x(x + 2) - 4 = x^2 + 2x - 4$. Result **$(x^2 + 2x - 4)/(x^2 - 4)$**.

⚠ Watch out!

× Wrong	✓ Right
$(x + 3)/(x + 2)$ can be simplified to $3/2$ by cancelling x.	You may cancel only factors, not terms: $x + 3$ and $x + 2$ have no common factor.
$1/a + 1/b = 2/(a + b)$.	$1/a + 1/b = (a + b)/(ab)$.
Forgetting restrictions after simplifying.	State them from the original expression.
Dividing: multiply by the first fraction's reciprocal.	Invert the SECOND fraction.

Practice 2.1

Level 1 — Basic

1. Simplify $(x^2 - 4)/(x - 2)$.
2. Simplify $6x/(3x^2)$.
3. Add $1/x + 2/x$.

⚠ Watch out!

× Wrong	✓ Right
$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad + bc$.	$\det = ad - bc$.
$\det(A + B) = \det A + \det B$.	This is false in general. Only $\det(AB) = \det A \det B$.
A singular matrix has an inverse.	$\det = 0$ means there is no inverse.
$\det(kA) = k \det A$ for a 2×2.	$\det(kA) = k^2 \det A$ for a 2×2.

Practice 4.1

Level 1 — Basic

1. Find $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix}$.
2. Find $\begin{vmatrix} 5 & 2 \\ 5 & 2 \end{vmatrix}$.
3. Is $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ singular?

Level 2 — Standard

4. Find $\begin{vmatrix} 3 & -1 \\ 2 & 5 \end{vmatrix}$.
5. Find k if $\begin{vmatrix} k & 4 \\ 2 & 3 \end{vmatrix} = 0$.
6. If $\det A = 3$, find $\det(2A)$ for a 2×2 matrix.

Level 3 — Challenge

7. Solve $\begin{vmatrix} x & 2 \\ 3 & x - 1 \end{vmatrix} = 0$.
8. If $\det A = 5$ and $\det B = -2$, find $\det(AB)$ and $\det(A^{\mathsf{T}}B)$.

4.2 Minors and Cofactors

Key ideas

- The **minor** M_{ij} of an entry a_{ij} is the determinant of the smaller matrix obtained by deleting row i and column j.
- The **cofactor** is $\mathbf{C_{ij} = (-1)^{i+j} M_{ij}}$. The sign pattern for a 3×3 matrix is $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$.
- The determinant is the sum of (entries of any row or column) × (their cofactors): $\mathbf{\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + a_{i3}C_{i3}}$. Expand along the row or column with the **most zeros**.
- The **cofactor matrix** has the cofactors in place of the entries; the **adjoint** (adjugate) is its **transpose**.

6.6 Applications of Transformations

Key ideas

- Transformations are used in **design** (patterns, logos), **computer graphics** (moving, flipping, turning shapes) and **geometry** (proving congruence, finding symmetry).
- **Symmetry:** a figure has **line symmetry** if a reflection maps it to itself and **rotational symmetry** if a rotation (less than 360°) maps it to itself.
- **To describe a transformation fully:** translation → vector; reflection → mirror line; rotation → centre, angle and direction.
- To identify a transformation, compare the coordinates of an object and its image.

Example 6.26 — Identify

A(1, 2), B(3, 5) are mapped to A′(−1, 2), B′(−3, 5). Describe the transformation.

Solution. x changes sign and y stays: a **reflection in the y-axis**.

Example 6.27 — Identify 2

P(2, 1) → P′(4, 3) and Q(5, 4) → Q′(7, 6). Describe it.

Solution. Both change by (2, 2): a **translation by (2, 2)**.

Example 6.28 — Symmetry

How many lines of symmetry and what rotational symmetry has a square?

Solution. 4 lines of symmetry and rotational symmetry of **order 4** (90°).

Example 6.29 — Exam Booster · Design

A flag triangle A(0, 0), B(2, 0), C(1, 2) is reflected in the y-axis and the pair is then translated by (6, 0). Write the final coordinates of the reflected triangle.

Solution. Reflected: (0, 0), (−2, 0), (−1, 2). After translation: **(6, 0), (4, 0), (5, 2)**.

Example 6.30 — Exam Booster · Rotation symmetry

What is the order of rotational symmetry of a regular hexagon?

Solution. 6 (every 60°).

Key Formula Sheet

Learn these by heart. Cover the right-hand side and write the formula from memory.

Unit 1 · Functions

$(f \circ g)(x) = f(g(x))$

f^{-1}: swap x and y, solve for y

$f(f^{-1}(x)) = x$

Profit = revenue − cost

Unit 2 · Rational

Factorise, then cancel factors

Check every answer for extraneous roots

Vertical asymptote: denominator = 0

Horizontal: compare degrees

Work: $1/a + 1/b = 1/t$

Unit 3 · Matrices

A is $m \times n$, B is $n \times p \Rightarrow$ AB is $m \times p$

$(AB)^T = B^T A^T$

$[[a,b;c,d]]^{-1} = (1/(ad - bc))[[d,-b;-c,a]]$

$AX = B \Rightarrow X = A^{-1}B$

Unit 4 · Determinants

$|a,b;c,d| = ad - bc$

$\det(AB) = \det A \cdot \det B$

$A^{-1} = \text{adj } A / \det A$

Cramer: $x = D_x / D$

Area $= \frac{1}{2}|\det [[x_1,y_1,1;x_2,y_2,1;x_3,y_3,1]]|$

Unit 5 · Vectors

$|a| = \sqrt{(a_1^2 + a_2^2 + a_3^2)}$

$a \cdot b = a_1b_1 + a_2b_2 + a_3b_3 = |a||b|\cos\theta$

$|a \times b| = |a||b| \sin\theta$

$AB = B - A$; midpoint $= (A + B)/2$

$W = F \cdot d$

Unit 6 · Transformations

Translation: $(x + a, y + b)$

x-axis: $(x, -y)$; y-axis: $(-x, y)$

$y = x$: (y, x); $y = -x$: $(-y, -x)$

Rotation 90°: $(-y, x)$; 180°: $(-x, -y)$

Area factor $= |\det M|$

Unit 7 · Statistics

$\bar{x} = \Sigma fx / \Sigma f$

$\sigma^2 = \Sigma f(x - \bar{x})^2 / \Sigma f$

$IQR = Q_3 - Q_1$

$z = (x - \bar{x})/\sigma$

Median $= L + ((n/2 - F)/f)w$

Unit 8 · Probability

$n!$; $P(n,r) = n!/(n - r)!$

$C(n,r) = n!/(r!(n - r)!)$

$(a + b)^n$: T(r+1) = C(n,r) a^(n−r) b^r

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A') = 1 - P(A)$

16. If $A = \begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 \\ 4 & 1 \end{bmatrix}$, the entry in row 1, column 1 of AB is

A. 12 B. 28 C. 8 D. 16

17. If A is 3 × 2 and B is 2 × 5, then AB has order

A. 2 × 2 B. 3 × 2 C. 5 × 3 D. 3 × 5

18. Solve x + y = 11 and x − y = −1. Then x =

A. 11 B. 6 C. 5 D. −1

19. The matrix $M = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ has det M = 1. The entry in row 1, column 1 of M^{-1} is

A. 1 B. −2 C. 2 D. 1/2

20. If $A = \begin{bmatrix} 4 & 2 & 3 \\ 4 & 3 & 2 \end{bmatrix}$, then A^T has order

A. 2 × 2 B. 3 × 2 C. 3 × 3 D. 2 × 3

21. The matrix $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ is singular when its determinant is 0. Its determinant is

A. 2 B. 1 C. 4 D. 3

22. The rank of $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is

A. 2 B. 1 C. 0 D. −1

23. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 4 \\ 4 & 1 \end{bmatrix}$, the entry in row 1, column 1 of AB is

A. 5 B. 4 C. 10 D. 16

24. The determinant of $\begin{bmatrix} 4 & 1 \\ 6 & 5 \end{bmatrix}$ is

A. 26 B. −26 C. 19 D. 14

25. The determinant of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 0 \end{bmatrix}$ is

A. 2 B. −1 C. 1 D. −2

26. The area of the triangle with vertices (0, 0), (2, 0) and (0, 3) is

A. 3 B. 5 C. 5/2 D. 6

27. Using Cramer's rule for x + y = 2, x − y = 0: D = −2 and D_x = −2. Then x =

A. 2 B. 7 C. 1 D. −1

28. For a 3 × 3 matrix A with det A = 2, det(2A) =

A. 12 B. 8 C. 4 D. 16

29. $\begin{bmatrix} k & 3 \\ 2 & 4 \end{bmatrix}$ is singular when k =

A. 2/3 B. 6 C. 3/2 D. 4